An Analytical Series DC Motor Model from Experimental Test Data

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Introduction

DC Motors are one of the most common mechatronic actuators in use today in a variety of applications. They are important for electromechanical servo systems, drivers for battery powered appliances and tools as well as electric vehicles. Both brushless DC motors and wound DC motors are common in electric and hybrid vehicles. The Series wound DC motor is commonly used for high torque vehicle applications. The literature has many papers discussing permanent magnet DC motors but a very limited number of publications on analytical models for series wound DC motors, especially motor models that fit test data for series wound DC motor available in the market place.

An analytical model for a series wound DC motor is developed here based on physical principles including energy conservation. The model developed will be compared with models developed by other investigators. Available commercial test data for two series motors will be used to find model parameters for the analytical model and the accuracy of these models evaluated against the original test data. The models developed display excellent accuracy well within the accuracy of the test data available. Typical model rms deviation from test data is under 2% for the commercial series wound DC motors evaluated.

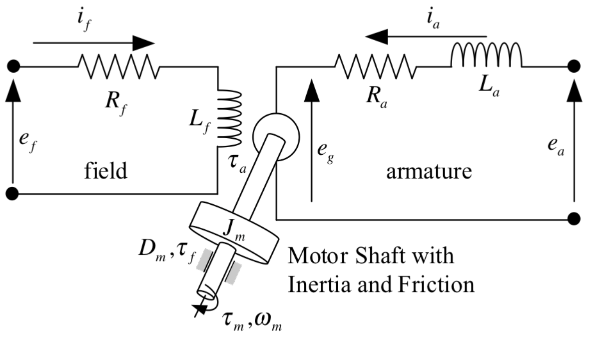

Figure 1: Schematic of a DC Motor with Separate Field and Armature Windings (adapted [1])

The Model

Redefine the field and armature variables using a series connection

$$\begin{aligned} e_m &= e_f + e_a \\ i_m &= i_f = i_a \end{aligned} \tag{1}$$

Using the series connection, the motor model for a series wound DC motor is

$$\begin{aligned} e_m &= R_m i_m + L_m \frac{di_m}{dt} + K_m \omega_m i_m^{\alpha} \\ \tau_m &= K_m i_m^{(1+\alpha)} - d_m \omega_m - \tau_d - J_m \frac{d\omega_m}{dt} \end{aligned} \tag{2}$$

Defining a model squared error

$$E^2 = \sum_i \left[\begin{array}{c} \left[1 - \dfrac{\left(R_m i_m + K_m \omega_m i_m^{\alpha} \right)}{e_m} \right]_i^2 + \\ \left[1 - \dfrac{\left(K_m i_m^{(1+\alpha)} - D_m \omega_m - \tau_d \right)}{\tau_m} \right]_i^2 \end{array} \right] \tag{3}$$

allows a) the fitting of parameters to experimental data and b) the solution of the nonlinear equations. These solutions fit the experimental measurement exceptionally well. In the case of a Model 140-01 5.5" motor from Advanced DC Motors, Inc, the fit is within an rms error of 1.4%.

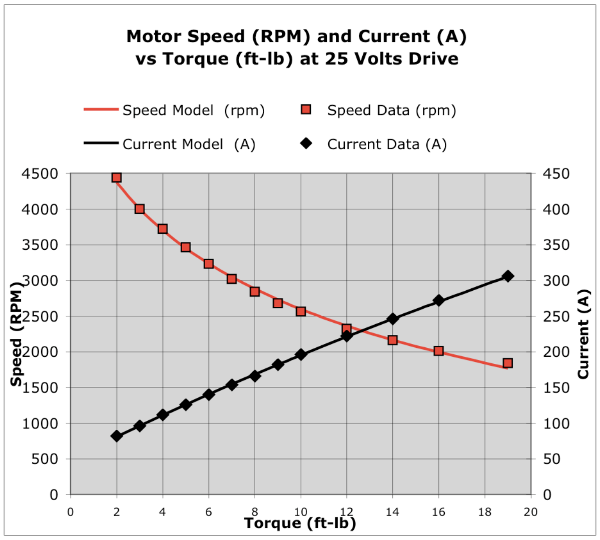

Figure 2: Modeled & Measured Responses vs Torque (Model 140-01 5.5", Advanced DC Motors, Inc)

References

[1] Phillips, C.L. and Harbor, R.D, *Feedback Control Systems*, 4th ed., Prentice Hall, Upper Saddle River, N.J.,pp. 38-43, 2000